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Keg impact damage to fibre-reinforced paving flags

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The incidence of cracked and unserviceable paving slabs (or flags) in pedestrian walkways is high, often due to the fact that plain concrete flags cannot withstand impact loading arising from unloading of materials during delivery. Subsequently, lack of maintenance eventually leads to sufficient damage such that a trip hazard develops, creating a public safety issue. For example, in the delivery of beer kegs to hotels, restaurants, bars and nightclubs, the use of a buffer bag to absorb the impact energy of a falling cask is often ineffective, resulting in clearly recognisable cracking patterns in the flags. This paper will describe the crack patterns that arise from cask impact loading on pavements. It will outline how the addition of macro polypropylene or steel fibres into the concrete mix used for paving flags affects the impact response. The extent to which the fibres provide residual post-cracking strength to resist the subsequent vehicle or other loading will also be discussed. Further, a yield-line analysis will reveal the reduced dependency on the contribution of the subgrade to the response when the ductile behaviour provided by the fibres is taken into account.

NOTATION

A_i	area of the ith planar region over which the subbase is in compression
E_c	modulus of elasticity for concrete
h	slab thickness
k	modulus of subgrade reaction
l_i	length of a yield line
L	radius of relative stiffness
M	plastic moment capacity
P	applied load
PFRC	polypropylene fibre-reinforced concrete
SFRC	steel fibre-reinforced concrete
$\delta_{av\,i}$	average vertical deflection of area A_i
δ	peak deflection under load
θ_i	angle between any two planar regions that form a yield line
ν	Poisson's ratio

1. INTRODUCTION

An inspection of the condition of pavements comprising paving slabs (or flags) often provides evidence of damage, particularly cracking and, occasionally, excessive deformation leading to a potential trip hazard for pedestrians[1] (Figure 1(a)). Consideration of the causes of such a failure in their functionality reveals that poorly prepared subbases combined with excessive loading are the most common causes. In particular, it is frequently the case that outside places of entertainment (such as hotels, restaurants, bars or nightclubs) there are a number of cracked flags, often with similar crack patterns. Further study reveals that the process of delivery of beer kegs involves a process of dropping kegs (typically from 1 m) onto a 40 cm square buffer bag, the purpose of which is to absorb the energy of the keg impact on the concrete pavement (Figure 1(b)). The deterioration of these cork-filled bags, or their omission altogether, however, gives rise to a sufficiently severe impact event on the thin paving flag which

(a)

(b)

Figure 1. Impact of beer keg on paving flag: (a) typical impact damage and (b) with buffer bag[2]

causes an initial crack to appear. Subsequent loading from further impacts or from delivery trucks and/or other vehicles parking illegally on the footpath, possibly exacerbated by the presence of inadequate subbase compaction during construction, is sufficient to progress the damage to such an extent that a trip hazard develops which is a safety risk for members of the public. Indeed, there is some evidence that local authorities have had to defend the construction and maintenance of the pavements in the courts due to personal injury claims arising from a fall caused by tripping.[3]

A standard construction detail exists for laying a paving system[4] (Figure 2) and guidelines for its construction and its condition status (in term of surface regularity) are well defined.[5] Either an out-of-flatness of more than 10 mm on a 3 m straight edge or a difference in level between two adjacent flags of more than 2 mm constitutes non-compliance.

1.1. Impact on concrete slabs on grade

The nature of an impact on a concrete slab can be classified in several different ways: the speed of impact; the damage to the impactor and the type of defect which arises in the target. As a keg falls under gravity from a relatively small drop height, the speed of impact is classed as low velocity. In the case of a keg impact on a concrete flag, the keg is manufactured using aluminium and has a reinforced lip at both ends. As a consequence of which, little, if any, permanent damage is experienced by the impactor. The impact is therefore classed as hard. Since concrete is relatively weak in tension and is a brittle material, damage due to severe impact on unreinforced concrete is high, giving rise to extensive cracking. This is despite the apparent increase in the strength properties at such high strain rates of loading.[6] Typically, thin concrete slabs under hard impacts can experience local damage. In order of increasing severity, damage includes top surface spalling, surface penetration, back face scabbing, shear plug formation on the back face and ultimately full perforation.[7,8] In addition, depending on geometry and support conditions, global flexural cracking can occur. Radial sagging cracks propagate on the bottom face emanating from the location of impact, usually transmitted and reflected in the top surface. If the slab is sufficiently wide, circumferential cracking on the top surface can occur due to the hogging moments that arise remote from the impact location. In the case of a paving flag on grade, the response to the impact load is further complicated by the resistance offered by the subbase, where deformation and crack propagation is ameliorated by the soil/subbase system.

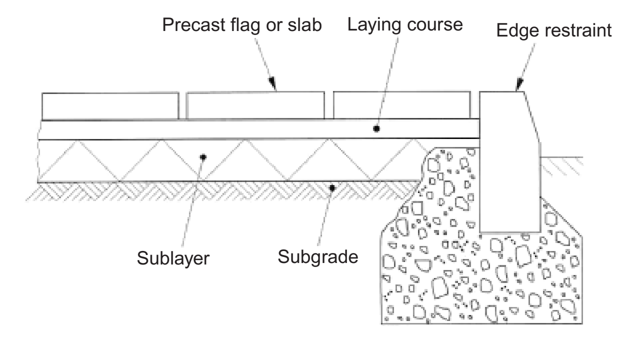

Figure 2. Typical detail of a pavement system for pedestrian walkways[4]

1.2. Use of fibres in concrete slabs on grade

A variety of fibre types can be used in concrete to enhance its properties. In particular, for large slabs on grade, short polypropylene fibres are effective in preventing plastic shrinkage cracking and longer steel fibres are commonly used to prevent long-term drying shrinkage cracking and to reduce joint spacing.[9] Toughness in flexure and impact resistance are also enhanced through the inclusion of fibres,[10,11] particularly long steel or macro polypropylene fibres. The fibres rely on good bond with the cement matrix, brought about by fibrillation or hooked ends and a high aspect ratio (length to equivalent diameter).

Under applied loading, the fibres at normal dosage rates have little effect on the compressive strength but bind the concrete together when the initial cracks occur, preventing crack propagation and widening.[12,13] When tested in flexure using displacement control,[14] a suspended fibre-reinforced slab has better toughness than its plain concrete counterpart and failure depends on the fibre type: steel fibres fail by pull-out while polypropylene fibres can fail by the fibres failing in tension. By virtue of its high elastic modulus, steel fibres are better at resisting crack growth.

For paving flags, the weight of a standard $60 \times 60 \times 6{\cdot}3$ cm flag (54 kg) is designed to be able to be lifted safely into place by two operatives. If improved performance is to be demanded due to the problem of cracking on impact or overloading, increasing the depth or width of the flag (thereby activating the hogging resistance) may enhance its impact properties. However, practical weight lift restrictions for manual labour prevent such a development in the product. It is therefore worthwhile to consider the alternative of using fibres to enhance the impact resistance of paving flags, such that they become less likely to develop into trip hazards in the future.

2. EXPERIMENTAL PROGRAMME

A series of testing programmes were put in place to establish the performance of paving flags that had been reinforced using two different fibre types. A typical base mix used by a local concrete producer of standard paving flags was adopted in each test programme (Table 1), with the single exception that a plasticiser was added to enable compaction using a vibrating table in the laboratory and not by vibrator compactor as used on the original dry mix in a factory.

A control plain concrete and polypropylene fibre-reinforced concrete (PFRC) and steel fibre-reinforced concrete (SFRC) mixes were made, from which four flags of each type were manufactured. The fibres used in the different mixes were fibrillated macro polypropylene fibres of length 45 mm (with equivalent diameter = 1 mm) at a dosage rate of 5 kg/m^3 and 50 mm hooked 45/50 Dramix steel fibres at a dosage rate of 50 kg/m^3 (Figure 3).

3. PRELIMINARY TESTS ON SUSPENDED PAVING FLAG AND SUBBASE STIFFNESS

3.1. Suspended flag tests

In order to characterise the impact response of plain and fibre-reinforced suspended paving flags, preliminary static flexural

Constituent	Quantity: kg/m^3
20 mm crushed limestone aggregate	340
10 mm crushed limestone aggregate	510
Medium sand	930
CEM1 N cement	370
GGBS	70
Sikament 10 plasticiser	1·8
Water	180
(Polypropylene fibre)	5·0
(Steel fibre)	50·0

Table 1. Constituents of the base mix, with fibres

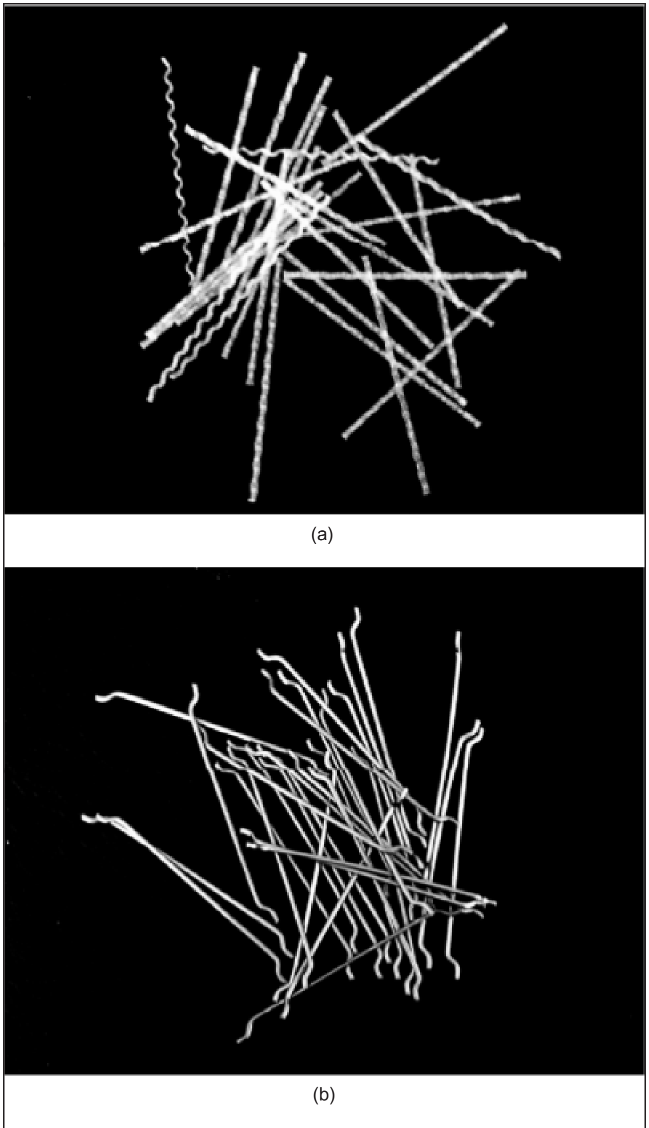

Figure 3. (a) Fibrillated polypropylene and (b) hooked steel fibres

tests were conducted in advance of the main impact testing programme.

For the flexural tests,[14] an internal reaction rig was bolted through a 900 mm deep heavily reinforced structural testing floor (Figure 4). A servo-controlled actuator was used to apply a displacement at a given rate, using a calibrated load cell to record the resistance offered by the flag.

The load–displacement responses of the three flag types are given in Figure 5, from which it may be observed that the plain concrete response (line a) was brittle, as expected. The PFRC flags had some residual load capacity after first crack (line b), but the toughness (the area under the post-cracking load–displacement curve) was low. The SFRC flags had a much better performance (line c), as expected. There is little

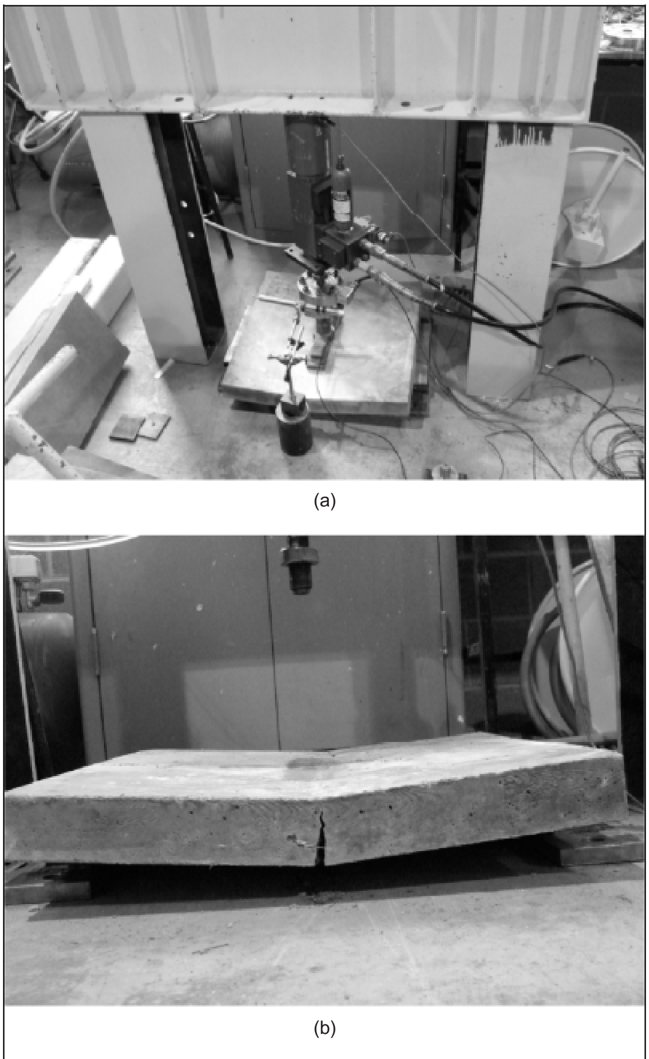

Figure 4. (a) Suspended simply supported paving flags in reaction frame with actuator and (b) failed fibre-reinforced flag with some residual strength due to fibre pull-out resistance

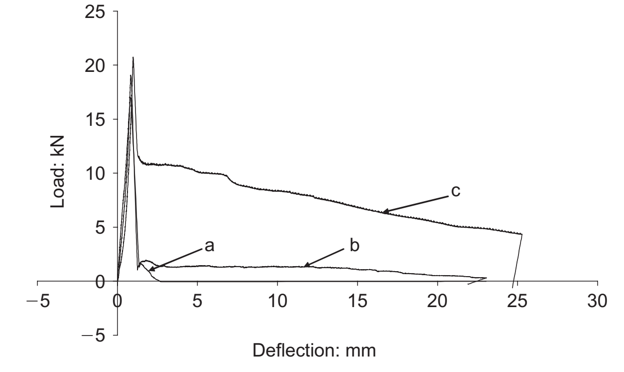

Figure 5. Load–deflection response of the three concrete flag types in a static simply supported test[2]

difference in the peak values, which are largely determined by the maximum flexural tensile capacity of concrete alone. For the fibre-reinforced flag tests, the significant drop off in load-carrying capacity when the concrete cracks is attributable to the fact that the fibres themselves cannot carry a large load in tension at transfer. First, their total cross-sectional area in tension is relatively small and second, as they are distributed throughout the depth and are not orientated in the principal flexural stress direction (that is, horizontally), they are not fully efficient in the area they do provide.

From the test results for the peak residual loads (based on the four flag types tested), the plastic moment capacities of these flags can be calculated to be $M_{PFRC} = 0{\cdot}35$ kNm/m and $M_{SFRC} = 2{\cdot}7$ kNm/m. These figures will have relevance in this paper when calculating the post-impact load capacity using yield-line analysis.

A mechanism for evaluating the fibres' effectiveness in improving toughness is in the calculation of the area under the load–deflection curve, the so-called equivalent strength ratio. The values in Table 2 give these parameters, evaluated as the area under the load–deflection curve post-cracking up to 3 mm (known as $R_{e,3}$) and 9 mm ($R_{e,9}$) deflection[15] (Figure 6). These values have relevance when interpreting the impact response of the various flag types. Note that as the plain flags are brittle, their equivalent strength ratios are considered to be zero.

3.2. Plate test

In order to evaluate the axial stiffness of the compacted subbase under the paving flags and therefore to establish the modulus of subgrade reaction *k*, an axially loaded plate test (using a stiff 15 cm square steel plate) was conducted. The outcome of such a test depends primarily on the soil type, plate size and stiffness. The results (Figure 7) showed that for a scale model pavement made up in the laboratory with the prescribed materials and compactive effort, a *k* value of 0·12 N/mm^2/mm was derived, which was in the range expected.[16,17]

Units: kN mm	$R_{e,3}$	$R_{e,9}$
Plain	0	0
PFRC	4·2	12·6
SFRC	32·4	87·2

Table 2. Equivalent strength ratios for plain, PFRC and SFRC paving flags (in kN mm)

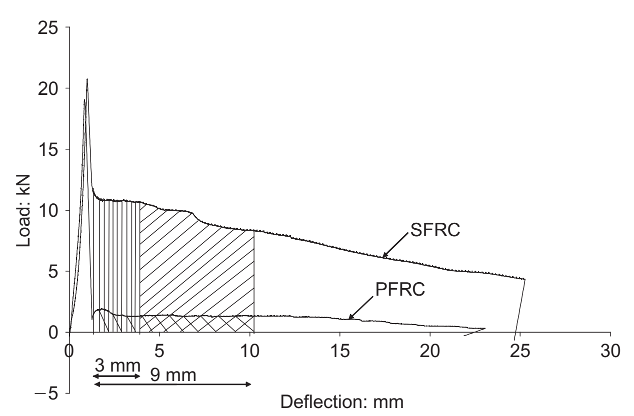

Figure 6. 3 mm and 9 mm toughness of PFRC and SFRC flags

4. CASK IMPACT

4.1 Test conditions

Several model pavements were assembled (Figures 8 and 9) for multiple impact and static testing. The subbase comprised a clause 804 aggregate[18] of 15 cm thickness, with a coarse sand

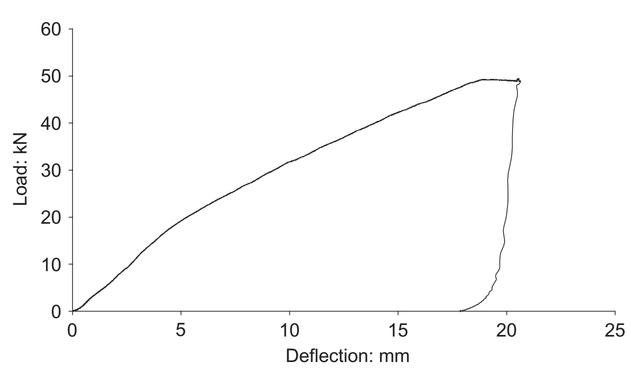

Figure 7. Typical load–deflection response in a plate-bearing test on the compacted subgrade


Figure 8. Compacting subgrade of model pavement


Figure 9. Impact of beer keg at 45° onto centre of paving flag

bedding layer of 2·5 cm thickness. While cement-bound materials or lean mixes are sometimes used for the bedding layer, the choice of a sand layer was the most conservative in terms of impact damage due to a lower k value. The compactive effort appropriate to the available 1800 kg/m^2 vibrator plate was 8 passes for both the subbase and the bedding layer. A jointing sand was used to fill the 2–3 mm gap between adjacent flags.

It should be noted that, in respect of the static and impact loading on grade, the test conditions in the laboratory were such that the model pavement lies on a single layer of 1·9 cm plywood sheet, which itself sits on a 90 cm deep heavily reinforced concrete floor. Clearly, this provides a more rigid subbase than is found in the field, where typically a firm soil base provides the formed surface for the compacted clause 804 material. Given the rigidity and density of the concrete flags (of thickness 6·3 cm) compared to the more energy-absorbing sand (2·5 cm thick) and aggregate (1·5 cm) layers, however, this necessary change to the boundary conditions under the subgrade is likely to yield reasonably realistic results. This is especially true as poorly compacted subbase and not subgrade failure is more likely to be a contributory factor to flag failure in practice.

The concrete flag size was 60 cm^2 $\times$ 6·3 cm thick. Each flag's footprint was tested only once: damaged flags were replaced with a dummy flag and jointing sand was restored between impact tests. During impact, the vertical movement of adjacent flags was monitored to ensure there was no disturbance to the surrounding subbase layer or flag.

The impact load was induced by an aluminium keg of dimensions 53·5 cm high and 42·0 cm in diameter, weighing 62 kg. Free drop heights under gravity varied from 0·25 m to 1·5 m in increments of 0·25 m. The keg orientation on impact was either vertical, horizontal or at a 45° angle. The orientation was fixed using a special bar sliding mechanism and a quick-release Bowden cable system.[19]

4.2. Results

The cask was repeatedly dropped at different orientations until a visible crack formed. Despite repeated drops of the various specimens from different heights onto the flags, there was no evidence of permanent plastic damage to the cask, justifying the classification of the impact as hard.

When the cork bag was used to absorb the energy of the impact, there was no evidence of the flag developing any cracks, even under 60 successive drops from a height of 1 m at the most onerous orientation. In some cases, however, under just one unprotected impact on the concrete flag, visible cracking occurred in the top surface of the flag.

Three distinctive patterns emerged depending on the keg orientation on impact, as shown in Figure 10. With the keg horizontal, impact caused a single central line crack to form (Figure 10(a)), with some evidence of spalling at locations where the ribs in the keg protruded out from the otherwise smooth cylindrical surface. With the keg vertical, the impact area was the circumference of the keg's end rim, as evidenced by a white circular scarring of the surface and by both

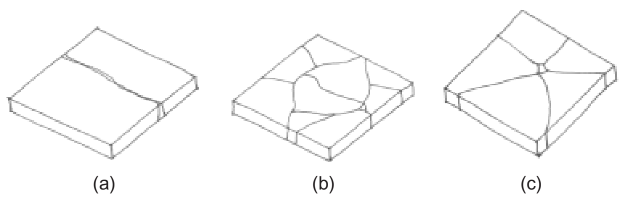

Figure 10. Crack patterns drawn using digital imaging and Autocad 2006: (a) horizontal, (b) vertical and (c) at 45°[20]

circumferential and tangential cracks developing (Figure 10(b)). The most severe impact was when the cask was impacted at 45°. Local spalling at the point of impact was evident when released from the greater drop heights, in addition to the characteristic star-shaped cracks with the apex at the centre of the flag (Figure 10(c)). In some cases, a minor bifurcation of the crack took place close to the edge of the flag.[19]

After the first visible crack occurred, a high-definition digital camera was positioned directly overhead to record patterns and crack widths. The digital image was converted into an Autocad 2006 drawing and an imaging software package, Image J, enabled crack widths to be estimated optically.

Table 3 lists the number of drops required to cause a crack to develop with plain concrete flags. Note that, in most cases, a small number of drops is all that is required to develop a crack. In the case of the vertical drop from a low height, however, more than 20 drops were required, reflecting the larger and non-central impact footprint for this load case.

Subsequently, upon repeated impacts, the extent of the damage propagation was also recorded. After five impacts, the crack widths and spalling damage were recorded, as listed in Table 4. It may be observed that, broadly, impact damage increases with drop height in increasing magnitude with vertical, horizontal and inclined orientation.

In several separate experiments in which single flags were tested without any adjacent flags in place, the damage on repeated impact increased disproportionately due to the absence of any lateral restraint.[20] This emphasised the importance of the passive confinement offered by the jointing sand and the surrounding flags in restricting the growth of a crack once formed. This confinement is important in resisting rapid crack growth and damage development.

Similarly, if the bedding layer was not properly compacted,

Orientation	Drop height: m			
	0·25	0·5	1	1·5
Horizontal	2	2	2	1
Vertical	>20	7	3	1
Inclined	1	2	2	1

Table 3. Number of drop occurrences to initiate first visible crack[20]

Orientation	Drop height: m				Drop height: m			
	0·25	0·5	1	1·5	0·25	0·5	1	1·5
	Crack width: mm				Spall width: mm			
Horizontal	1·42	1·63	1·65	2·62	5·64	13·16	13·16	8·45
Vertical	—	—	1·34	1·74	—	2·69	2·33	8·12
Inclined	2·24	1·90	1·26	12·08	6·90	9·43	10·67	16·70

Table 4. Damage recorded as crack and spall widths following five drops from a 1 m height[20]

tests showed that damage was much more likely and severe, emphasising the importance of an integral subbase in mitigating damage.[20]

It took at least three 1 m high, 45° impacts on the SFRC flags to obtain a visible crack, although on lifting the flag afterwards a crack had fully developed underneath.[2] In fact, considerable surface spalling was evident before a clearly visible crack occurred. This indicated that the steel fibres were somewhat more efficient at preventing cracks developing on the surface on impact. This is not true of the PFRC flags which, like the plain flags, had a visible crack on the first impact. This is not surprising, given the trends in the first crack and residual loads in the static load–deflection responses shown in Figure 6.

4.3. Wider slabs

In exploring the options for improving the performance of slabs under impact, a small number of tests were undertaken on wider slabs.[21] This was done so that the benefit of mobilising the circumferential cracks in the top surface under hogging moments at some distance from the impact location could be evaluated.[17]

With a drop height of 1·5 m at the most damaging orientation of 45° and a slab size of 1·75 m square with 70 mm depth, circumferential cracks were developed under repeated impacts (Figure 11). The diameter of the actual circular cracks ranged from 315 mm (SFRC) to 610 mm (plain RC), which compared reasonably well with the predicted diameter of 470 mm using the following crude expression for the radius of relative stiffness[17] (which incidentally takes no account of reinforcing type)

$$L = \left[\frac{E_c h^3}{12(1-v^2)k}\right]^{0.25} \quad (1)$$

where E_c is the modulus of elasticity for concrete, h is the slab thickness, ν is Poisson's ratio and k is the modulus of subgrade reaction.

Figure 11. Circumferential crack due to hogging moments on wider slab impact test[21]

The extent of the radial cracking for this case is listed in Table 5, where it may be observed that the SFRC slabs were the most effective at restricting crack widths. Note also that a slab containing conventional steel mesh (A 393 fabric) was also included in this study and performed no better than the PFRC slabs. It should also be noted that in lifting these slabs, a significant amount of back face scabbing and a shear plug had developed unnoticed underneath the slab.

Two issues mitigated against increasing the slab width, however, as a solution for reducing keg impact damage. First, the extra strength of the slab overall meant that considerable damage was experienced by the lip of the keg: permanent damage was evident through flattening of the rim. The impact was now softer and there was a risk of the keg rupturing under extended use. Thus, a 65 kg solid steel billet had to be used to crack the majority of these slabs. Second, a slab of dimension sufficient to develop a hogging moment (estimated as 1·5 m square) was such that its weight would be over 300 kg, which would be impractical for two operatives to lift into place. For these practical reasons, the fibre solution to damage mitigation was preferred to changing the geometry.

5. RESIDUAL LOAD-CARRYING CAPACITY

5.1 After impact

While it may be concluded from the above that the presence of fibres is not as important a factor as drop height, keg orientation and flag dimensions in determining if a crack will occur, the contribution of fibres (steel fibres in particular) to crack propagation is significant. To illustrate this further, the post-cracking behaviour of paving flags will now be considered. The residual load capacity is important, just as much as lateral restraint and subgrade integrity, in determining the growth and progression of cracking and therefore in determining the potential for a trip hazard to develop.

Slab type	Average radial crack width: mm
Unreinforced slab	4·98
Mesh reinforced	2·64
PFRC	2·60
SFRC	0·40

Table 5. Average radial crack widths for wider paving slabs[2]

A new pavement was manufactured in the laboratory with the same specification as already described. The individual flags (unreinforced, polypropylene or steel reinforced) were each impacted by a keg dropped from 1 m at 45° orientation until a visible crack was evident.

A post-cracking static load of 40 kN, representing a standard axle load of a commercial vehicle, was applied to each flag in turn using a mobile test frame (Figure 12). The steel reaction rig allowed a monotonic force of up to 48 kN to be applied to a cracked flag, where reaction was provided through large bags of aggregate placed in two demountable skips on top of the steel frame, acting as kentledge. Load was applied using a 100 kN servo-controlled actuator in displacement control mode[14] at a rate of 4×10^{-6} m/s through a 6·0 × 6·0 × 2·0 cm steel plate. A calibrated load cell and a number of linear variable differential transformer (LVDT) transducers were used to monitor the load and displacements, respectively, including the uplift at the flag's corners and in adjacent flags (Figure 13).

5.2. Results

Typical output plots of the static displacement response with load for the three flag types are superimposed in Figure 14.

At 40 kN, values of central deflection were 3·44, 2·34 and 1·25 mm for the plain, PFRC and SFRC flags, respectively, when load–deflection curves were normalised to the origin (i.e. after some bedding-in had taken place). The SFRC flags had the highest residual stiffness and retained their stiffness up to a maximum applied load of 48 kN, restricted by the available kentledge. The response on unloading was also elastic with recovery of over 60%, indicating that further residual strength existed. In contrast, the response of the plain flags was governed by the subgrade response because no residual flag flexural strength existed. It may be observed in Figure 14 that the PFRC flag response is not dissimilar to the plain concrete, as might be expected from Figure 5.

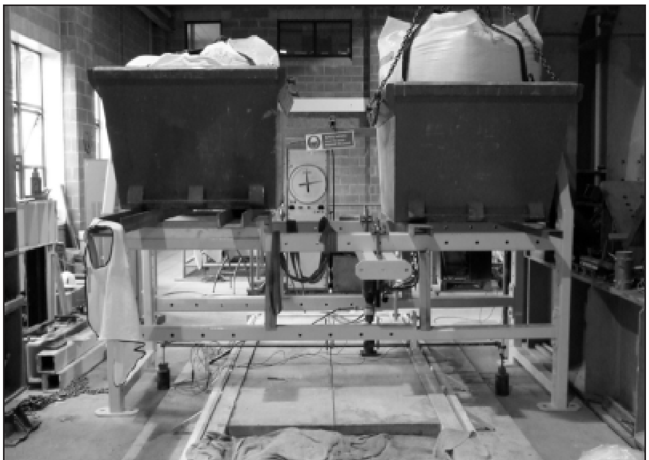
Figure 12. Reaction frame with skips filled of aggregate to provide kentledge for static testing of residual strength of impact-damaged paving flags


Figure 13. Use of LVDTs to measure uplift

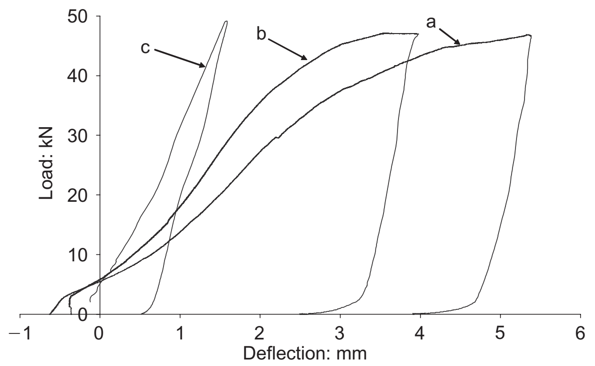

Figure 14. Load–deflection response of impact-damaged paving flags for: (a) plain concrete, (b) PFRC and (c) SFRC flags

A relevant feature of the response is the uplift which inevitably occurred in part of the free edges even when the load was removed, especially at the corners. Any difference in level greater than 2 mm between one flag and the next is deemed to be a trip hazard.[5] During loading, the peak uplifts (at the free edges) for the plain, PFRC and SFRC flags were 5·92, 5·50 and 0·5 mm respectively, with residuals of 3·97, 3·86 and 0·37 mm upon load removal. Only the SFRC flags therefore have sufficient residual stiffness and strength to prevent a trip hazard from developing. Despite having similar crack patterns prior to loading (see Table 6 for total crack lengths), the cracks widened at significantly different rates during loading. In particular, the SFRC flags were observed to have only just-visible cracks on the top surface with some evidence of opening up of a crack on the bottom surface (Figure 15). Note that the plain concrete flag collapsed on lifting as it had no residual strength and therefore no bottom surface crack pattern was obtainable.

It may be therefore be concluded from this experimental work: the presence of steel fibres arrests crack growth on static loading when a flag is already cracked by impact; the SFRC flag has considerable residual strength and therefore superior post-cracking toughness; top surface crack widths remain small (<0·1 mm); and the flags exhibit better elastic recovery so that corner lifting after the load is removed (<0·4 mm) is below the recognised trip hazard threshold. On the other hand, plain and PFRC flags exposed to post impact loading (e.g. parked vehicles) do develop a trip hazard, which would require urgent maintenance of the pavement.

6. YIELD-LINE ANALYSIS

A conventional yield-line analysis, where the rate of work done externally by the load is equated to the rate of internal

Units: mm	After keg impact	After 40 kN (top)	After 40 kN (bottom)	Total length of crack
Plain	0·75	1·16	—	1326
PFRC	0·25	0·28	2·36	1360
SFRC	<0·1	<0·1	0·96	1410

Table 6. Average crack widths and lengths (in mm) for different flags at various stages of loading[2]

Figure 15. Bottom surface cracking of the SFRC flag after application of 40 kN load

work done by the plastic moments developed in the slab,[22] cannot be employed in the case of plain paving flags on grade. This is not because there is no load-carrying capacity—the subgrade does provide some residual resistance—but because the plain concrete, once cracked, no longer has a moment-carrying capacity along the yield (cracked) lines. As soon as PFRC or SFRC is introduced with a post-cracking moment capacity (see Figure 6), however, then a formal yield-line analysis can be undertaken[23] with the advantage that the actual yield lines are known a priori from the results of the impact event. Furthermore, the edges of the flag are free and are known to uplift in some circumstances. This implies that while some energy is absorbed by the subbase in the work equation, it does not extend over the entire flag area. In fact, the lines of zero deflection are not predefined and have to be established through monitoring movement during loading. This adds a further complication to the work equation, thus

$$(\text{Work Done})_{\text{internal}} + (\text{Work Done})_{\text{subbase}} = (\text{Work Done})_{\text{external}} \tag{2}$$

or

$$M_{\text{p}} \sum_i \theta_i l_i + k \sum_i A_i \delta_{\text{av}\,i} = P\delta \tag{3}$$

in which θ_i is the angle between any two planar regions that form a yield line, l_i is the length of such a yield line, A_i is the area of the ith planar region over which the subbase is in compression and $\delta_{\text{av}\,i}$ is the average vertical deflection of area A_i given that the peak deflection δ occurs underneath the applied load P. The constants M_{p} and k are, respectively, the residual plastic moment capacity (established from the static residual load (Figure 6)) and the modulus of subgrade reaction (established from plate tests). A discrete linear elastic Winkler spring model is used to represent the modulus of subgrade reaction k of the compacted material under the paving flags.

The terms $\delta_{\text{av}\,i}$ and θ_i can be expressed in terms of δ, which is normally indeterminate in yield-line analysis, but which cancels on both sides of Equation 3. Furthermore, it is assumed initially and conservatively that the intersect of the yield line with the free edge represents the line of zero deflection in every case. This assumption will be evaluated presently.

A graphical method was used to determine the values of l_i, A_i, θ_i and $\delta_{\text{av}\,i}$ in the various regions of the flags bounded by the known yield lines and the free edges, where the relevant geometry was extracted from the photographic images of the cracks imported into Autocad 2006. If one selects the midpoint of a yield line to determine the angle θ_i between regions, then two possible cases arise.

(*a*) The section line (e.g. line 1–1 in Figure 16) intersects with both zero-deflection lines (AD and AB), to give

$$\theta_i = \xi_i \delta \left(\frac{1}{L_{1i}} + \frac{1}{L_{2i}} \right) \tag{4}$$

where ξ_i, $L_{1\,i}$ and $L_{2\,i}$ are defined in Figure 16.

(*b*) The section line (e.g. line 2–2 in Figure 17) intersects one zero-deflection line (AB) and a yield line (OC) to give

$$\theta_i = \delta \left(\frac{\xi_{1i}}{L_{i1}} + \frac{1 - (r_i/q_i)}{L_{2i}} \right) \tag{5}$$

where r_i and q_i are defined in Figure 17.

Proceeding through the calculations for each of the four tested PFRC and SFRC flags under a 45° orientation impact leads to the data listed in Table 7 in which the relative contribution from the yield line and the subgrade to the overall load capacity may be observed. While it is recognised that the number of flags tested and analysed is relatively small, some general trends in the behaviour can be observed.

The plain flags on grade rely entirely on the subbase material when impact cracks occur (which, as already stated, can be after just one accidental drop from 0·25 m). Therefore, the development of a trip hazard depends largely on the subbase integrity and so is very sensitive to whether the correct compaction has been executed during construction.

The presence of polypropylene fibres in the flag improves its capacity by approximately 33% above plain flags, and about

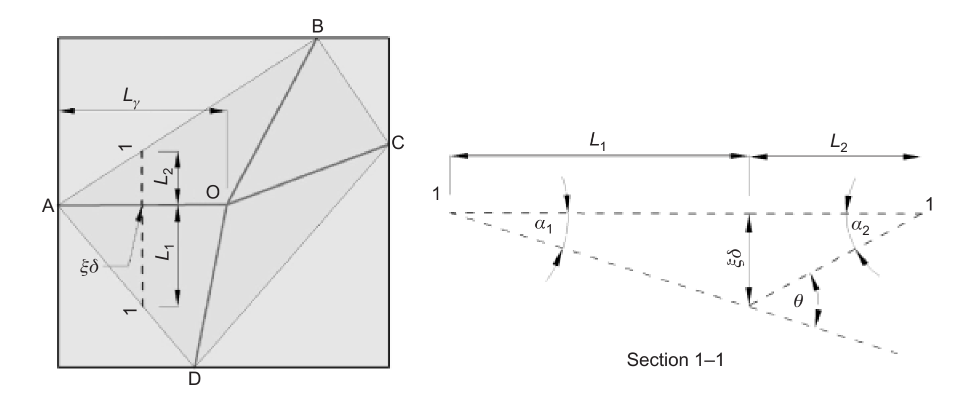

Figure 16. Establishing the angle between regions between the yield lines for case (a)

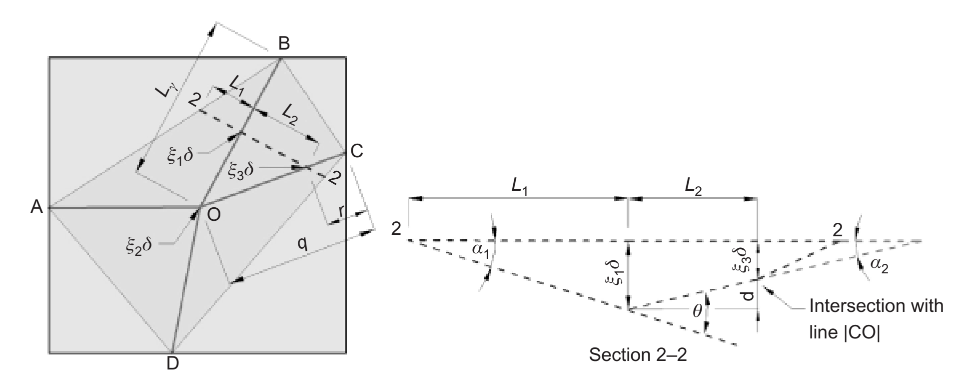

Figure 17. Establishing the angle between regions between the yield lines for case (b)

P: kN	Fibres	Subgrade	Total
PFRC	2·84	8·33	11·16
SFRC	21·61	8·30	29·91

Table 7. Average theoretical *P* load (in kN) of four flag specimens[2]

25% of the yield load is taken by the fibres along the yield lines. There is therefore still a heavy reliance on the subbase for load-carrying capacity.

Paving flags which are reinforced with long steel fibres do not crack visibly on a single 1 m drop, but do so on repeated drops. The theoretical plastic load-carrying capacity is increased by 260% over the plain flag capacity. Cracked SFRC flags appear to rely by over 70% on the residual load capacity in the yield lines to resist the applied static load and are therefore much less sensitive to poorly compacted subgrade. Furthermore, SFRC flags are much less likely to develop visible cracks, have a much smaller central deflection and much smaller uplift at the corners of the flag.

While in the cases considered the load taken by the subgrade is much less than its capacity (and remained linearly elastic), the flag's residual strength dictates the development of its rotational capacity. Serviceability limits are likely to dominate due to the development of a significant deformation and therefore a trip hazard.

6.1. Actual zero-deflection lines

The estimates of the contribution of the subgrade to the load capacity is conservative because the actual zero-deflection lines are not necessarily at the intersection of the yield lines and a free edge. Subsequent combined impact and static testing of flags with different impact orientations was undertaken with LVDT transducers placed on all corners and at the centres. From this, the actual zero-lines revealed the contribution of the subgrade to the capacity could be marginally higher, which results in minor adjustments to the statistics on subgrade dependency offered above.[24]

7. CONCLUSIONS

It has been shown that the unprotected impact of kegs on concrete paving flags causes particular patterns of cracking, even from relatively modest drop heights, and that the presence of fibres in the flags is unlikely to prevent this initial cracking from happening under repeated loading. It has also been shown, however, that long steel fibres have sufficient post-cracking stiffness and strength to prevent the deterioration of the cracked flag with time when subjected to static loading, typically from illegally parked vehicles. Polypropylene fibre-reinforced slabs, at typical dosage rates, cannot provide the same restraint and so continue to rely heavily on the degree of compaction of the subgrade to offer further load-carrying capacity post-cracking.

The method of yield-line analysis has been used to verify the experimental evidence and to make a case for recommending the inclusion of steel fibres in paving slabs in locations where they might be subject to impact loading, for example, during beer keg delivery. It may therefore be concluded that the inclusion of a dosage of 50 kg/m^3 of 50 mm hooked steel fibres is sufficient to prevent the degradation of paving slabs when damaged by impact from beer kegs. Furthermore, it is recommended that designers and specifiers strongly consider the specification of long medium dosage steel fibres as constituents in all paving flags which are prone to damage, particularly in areas of entertainment when beer kegs and other deliveries are prevalent. In this way, the potential for the development of a trip hazard, leading to a public safety issue, can be mitigated through the specification of steel fibre inclusion in paving flags by the responsible authorities.

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